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CONVOLUTION STRUCTURE OF FRACTIONAL QUATERNION FOURIER TRANSFORM

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ABSTRACT

The quaternion are a number system that extends the complex numbers. It uses in theoretical and applied branch of mathematics. The main applications of quaternion are filter design and color image processing. It has vital role in animation field.

The aim of our work is the convolution structure of Fractional Quaternion Fourier Transform is given which is useful in image processing. Also we have proved some basic properties like associative, distributive, linear, shifting for the convolution of Fractional Quaternion Fourier Transform.

KEYWORDS: Fourier transform, Fractional Fourier transform, fractional quaternion Fourier transform, quaternion.

INTRODUCTION

In 1929 the fractional Fourier transform has been introduced by Wiener, then it has been studied by Condon in 1937, Bargamann in 1961, de Bruijin in 1973 and it rediscovered by Namias in 1980, McBride & Kerr in 1987 and Mustard in 1987. Then Lohnmann in 1993, Ozaktas & Mendlovic in 1993-1994, Alieva in 1994, Almeida in 1994 has been studied the Fractional Fourier transform extensively in their research work [12]. Fractional Fourier transform has many applications like pattern recognition, optics, signal processing, watermarking , cepstrum analysis and in many other fields. [7,8]

Quaternion

The ideas of this calculus of quaternion, as distinguished from its operations and symbols, are fitted to be of the greatest use in all parts of science.

The Quaternion algebra over R denoted by H is an associative, non commutative

$$H = \{q = q_0 + q_1\bar{\imath} + q_2\bar{\jmath} + q_3\bar{k}, q_0, q_1, q_2, q_3 \epsilon R\}$$

and multiplication laws:

$ij = -ji = k,\ jk = -kj = i,\ ki = -ik = j,\ i^2 = j^2 = k^2 = ijk = -1.$

The quaternion fractional Fourier transform is mainly used in object recognition, reconstruction, in color images.[3,5] . Quaternion play an important role in animation field because it compose rotation nicely and mainly it gives spherical interpolation.

Quaternion Fourier Transform convolution is useful to study hypoellipticity and to solve the heat equation in quaternion algebra framework. It is also used to find the solution of generalized heat equation is extension of solution of the classical heat equation [6].

In our previous work [9] we have been studied convolution theorem for two dimensional fractional Fourier transform. In this paper we discussed the definition of fractional quaternion Fourier transform, the convolution structure and some important properties of fractional quaternion Fourier transform.

I. Definitions

2.1 Two Dimensional Fractional Fourier Transform

The two-dimensional fractional Fourier transform with parameters α of $f(x,y)$ denoted by 2DFRFT{ $f(x,y)$} performs a linear operation, given by the integral transform. 2DFRFT

$\{f(x,y)\} = F_\alpha(\xi,\eta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)K_\alpha(x,y,p,q)dxdy$----(1)

where $K_\alpha(x,y,p,q) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{1}{2sin\alpha}[(x^2+y^2+p^2+q^2)cos\alpha-2(xp+yq)}$

$= C_{1\alpha}e^{iC_{2\alpha}[(x^2+y^2+p^2+q^2)cos\alpha-2(xp+yq)}$

where $C_{1\alpha} = \sqrt{\frac{1-icot\alpha}{2\pi}}$, $C_{2\alpha} = \frac{1}{2sin\alpha}$ $0 < \alpha < \frac{\pi}{2}$. ---(2)

2.2 Fractional Quaternion Fourier Transform

For any two dimensional quaternion function

$f(x,y) = f_r(x,y) + if_i(x,y) + jf_j(x,y) + kf_k(x,y)$ given by $f_r(x,y)$, $f_i(x,y)$, $f_j(x,y)$ and $f_k(x,y)$ are real, the fractional quaternion Fourier transform of $f(x,y)$is denoted by $FRF^{i,j}_{\alpha_1,\alpha_2}(p,q)$ &

$$FRF^{i,j}_{\alpha_1,\alpha_2}(p,q) = FRF^{i,j}_{\alpha_1,\alpha_2}\{f(x,y)\}$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} K^i_{\alpha_1}(x,p)f(x,y)K^j_{\alpha_2}(y,q)dxdy$$

$$K^i_{\alpha_1}(x,p) = \sqrt{\frac{1-icot\emptyset_1}{2\pi}}\, e^{\frac{i}{2sin\emptyset_1}[(x^2+p^2)cos\emptyset_1-2xp]},\ \emptyset_1 = \alpha_1\frac{\pi}{2}$$

$$K^j_{\alpha_2}(y,q) = \sqrt{\frac{1-icot\emptyset_2}{2\pi}}\, e^{\frac{i}{2sin\emptyset_2}[(y^2+q^2)cos\emptyset_2-2yq]},\ \emptyset_2 = \alpha_2\frac{\pi}{2}$$

II. Convolution structure for Fractional Quaternion Fourier transform

Theorem: For any real, scalar or complex signal $f(x,y)$ and convolution kernel $g(x,y)$ and

$h(x,y) = (f*g)(x,y)$

$$= \sqrt{\frac{1-icot\emptyset_1}{2\pi}}\sqrt{\frac{1-icot\emptyset_2}{2\pi}}\, e^{\frac{-i}{2}[(x^2cot\emptyset_1+y^2cot\emptyset_2)}\, \{e^{\frac{i}{2}[(x^2cot\emptyset_1+y^2cot\emptyset_2)}\, f(x,y) * g(x,y) \quad e^{\frac{i}{2}[(x^2cot\emptyset_1+y^2cot\emptyset_2)}\}$$

where, * is the Fractional Quaternion Fourier Transform convolution operator then

$F_{\alpha_1,\alpha_2}\{h(x,y)\}(p,q) = e^{\frac{-i}{2}[(p^2cot\emptyset_1+q^2cot\emptyset_2)}\, F_{\alpha_1,\alpha_2}\{f(t,s)\}F_{\alpha_1,\alpha_2}\{g(m,n)\}$

Proof-

$F_{\alpha_1,\alpha_2}\{h(x,y)\}(p,q)$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y)\, K_{\alpha_1,\alpha_2}(x,y,p,q)dxdy$$

$$= \sqrt{\frac{1-icot\emptyset_1}{2\pi}}\sqrt{\frac{1-icot\emptyset_2}{2\pi}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y)\, e^{\frac{i}{2}(x^2cot\emptyset_1+p^2cot\emptyset_1+y^2cot\emptyset_2+q^2cot\emptyset_2)}$$
$$e^{-i(xpcosec\emptyset_1+yqcosec\emptyset_2)}dxdy$$

$$= \sqrt{\frac{1-icot\emptyset_1}{2\pi}}\sqrt{\frac{1-icot\emptyset_2}{2\pi}}e^{\frac{i}{2}[(p^2cot\emptyset_1+q^2cot\emptyset_2)}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y)\, e^{\frac{i}{2}(x^2cot\emptyset_1+y^2cot\emptyset_2)}$$
$$e^{-i(xpcosec\emptyset_1+yqcosec\emptyset_2)}dxdy$$

From given

$$= \left(\sqrt{\frac{1-icot\emptyset_1}{2\pi}}\right)^2\left(\sqrt{\frac{1-icot\emptyset_2}{2\pi}}\right)^2 e^{\frac{i}{2}[(p^2cot\emptyset_1+q^2cot\emptyset_2)}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}(x^2cot\emptyset_1+y^2cot\emptyset_2)}$$
$$e^{-i(xpcosec\emptyset_1+yqcosec\emptyset_2)}\ \{\ e^{\frac{-i}{2}[(x^2cot\emptyset_1+y^2cot\emptyset_2)}\ [e^{\frac{i}{2}[(x^2cot\emptyset_1+y^2cot\emptyset_2)}\, f(x,y)*g(x,y)$$
$$e^{\frac{i}{2}[(x^2cot\emptyset_1+y^2cot\emptyset_2)}]\}dxdy$$

$$= \left(\sqrt{\frac{1-icot\emptyset_1}{2\pi}}\sqrt{\frac{1-icot\emptyset_2}{2\pi}}\right)^2 e^{\frac{i}{2}[(p^2cot\emptyset_1+q^2cot\emptyset_2)}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i(xpcosec\emptyset_1+yqcosec\emptyset_2)}$$

$\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\tilde{f}(t,s)\tilde{g}\ (x-t,y-s)dtds\}dxdy$ (As per our previous work[9])

$$= \left(\sqrt{\frac{1-icot\emptyset_1}{2\pi}}\sqrt{\frac{1-icot\emptyset_2}{2\pi}}\right)^2 e^{\frac{i}{2}[(p^2cot\emptyset_1+q^2cot\emptyset_2)}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i(xpcosec\emptyset_1+yqcosec\emptyset_2)}$$

$$\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,s)e^{\frac{i}{2}[(t^2cot\emptyset_1+s^2cot\emptyset_2)}g\ (x-t,y-s)\ e^{\frac{i}{2}[(x-t)^2cot\emptyset_1+(y-s)^2cot\emptyset_2}dtds\}dxdy$$

$$= \left(\sqrt{\frac{1-icot\emptyset_1}{2\pi}}\sqrt{\frac{1-icot\emptyset_2}{2\pi}}\right)^2 e^{\frac{i}{2}[(p^2cot\emptyset_1+q^2cot\emptyset_2)}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i(xpcosec\emptyset_1+yqcosec\emptyset_2)}$$
$$f(t,s)g\ (x-t,y-s)e^{\frac{i}{2}[(t^2cot\emptyset_1+s^2cot\emptyset_2)}\ e^{\frac{i}{2}[(x^2cot\emptyset_1-2xtcot\emptyset_1+t^2cot\emptyset_1)}\ e^{\frac{i}{2}[(y^2cot\emptyset_2-2yscot\emptyset_2+s^2cot\emptyset_2)}dtds\}dxdy$$

Putting $x-t=m,\ \ y-s=n$

$x=m+t,\ \ y=n+s$

$$= \left(\sqrt{\frac{1-icot\emptyset_1}{2\pi}}\sqrt{\frac{1-icot\emptyset_2}{2\pi}}\right)^2 e^{\frac{i}{2}[(p^2cot\emptyset_1+q^2cot\emptyset_2)}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i[(m+t)pcosec\emptyset_1+(n+s)qcosec\emptyset_2]}$$
$$f(t,s)g(m,n)\ \ e^{\frac{i}{2}[(t^2cot\emptyset_1+s^2cot\emptyset_2)}\ e^{\frac{i}{2}[(m+t)^2cot\emptyset_1-2(m+t)tcot\emptyset_1+t^2cot\emptyset_1)}$$
$$e^{\frac{i}{2}[(n+s)^2cot\emptyset_2-2(n+s)scot\emptyset_2+s^2cot\emptyset_2)}dmdndtds$$

$$= \left(\sqrt{\frac{1-icot\emptyset_1}{2\pi}}\sqrt{\frac{1-icot\emptyset_2}{2\pi}}\right)^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,s)g(m,n)\ \ e^{\frac{i}{2}(t^2+p^2)cot\emptyset_1-itpcosec\emptyset_1}\ e^{\frac{i}{2}(m^2cot\emptyset_1-impccosec\emptyset_1)}$$
$$e^{\frac{i}{2}(s^2+q^2)cot\emptyset_2-isqcosec\emptyset_2}\ \ e^{\frac{i}{2}(n^2cot\emptyset_2-inqccosec\emptyset_2)}dmdndtds$$

$$= \left(\sqrt{\frac{1-icot\emptyset_1}{2\pi}}\sqrt{\frac{1-icot\emptyset_2}{2\pi}}\right)^2 e^{-\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,s)g(m,n)$$

$e^{\frac{i}{2}[(t^2+p^2)cot\emptyset_1+(s^2+q^2)cot\emptyset_2]}\ e^{-i[tpcosec\emptyset_1+sqcosec\emptyset_2]}\ e^{\frac{i}{2}[(m^2+p^2)cot\emptyset_1+(n^2+q^2)cot\emptyset_2]}$

$e^{-i[mpcosec\emptyset_1+nqcosec\emptyset_2]}dmdndtds$ --------------------(3.1)

$= e^{-\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,s)\,K_{\alpha_1,\alpha_2}(t,s,p,q)dtds$

$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(m,n)\,K_{\alpha_1,\alpha_2}(m,n,p,q)dmdn$

$= e^{\frac{-i}{2}[(p^2cot\emptyset_1+q^2cot\emptyset_2)}\ F_{\alpha_1,\alpha_2}\{f(t,s)\}F_{\alpha_1,\alpha_2}\{g(m,n)\}$--------------------------(3.2)

III. Some Basic Properties of Quaternion Convolution

3.1 Linearity property

Prove that

(i) $(A_1f + A_2g) * h = A_1(f * h) + A_2(g * h)$
(ii) $h * (A_1f + A_2g) = A_1(h * f) + A_2(h * g)$
where $A_1, A_2 \epsilon H$

Proof-
(i)Consider
$LHS = (A_1f + A_2g) * h$

$= \beta * h \qquad (\because \beta = (A_1f + A_2g))$

By using (3.1) and (3.2)
$LHS = \beta * h$

$= e^{\frac{-i}{2}[(p^2cot\emptyset_1+q^2cot\emptyset_2)}\ F_{\alpha_1,\alpha_2}\{\beta(t,s)\}F_{\alpha_1,\alpha_2}\{h(m,n)\}$

$= e^{-\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{\beta(t,s)\}\,K_{\alpha_1,\alpha_2}(t,s,p,q)dtds$

$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{h(m,n)\}\,K_{\alpha_1,\alpha_2}(m,n,p,q)dmdn$

$= e^{-\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{[A_1f(t,s) + A_2g(t,s)]\}\,K_{\alpha_1,\alpha_2}(t,s,p,q)dtds$

$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{h(m,n)\}\,K_{\alpha_1,\alpha_2}(m,n,p,q)dmdn$-----------------(1)

Now we consider
$RHS = A_1(f * h) + A_2(g * h)$
By using (3.1) and (3.2)

$= e^{\frac{-i}{2}[(p^2cot\emptyset_1+q^2cot\emptyset_2)}[A_1\{F_{\alpha_1,\alpha_2}(f * h)\} + A_2\{F_{\alpha_1,\alpha_2}(g * h)\}]$

$= e^{-\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)}\{A_1[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,s)\,K_{\alpha_1,\alpha_2}(t,s,p,q)dtds\ \ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(m,n)\,K_{\alpha_1,\alpha_2}(m,n,p,q)dmdn]$

$A_2[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(t,s)\,K_{\alpha_1,\alpha_2}(t,s,p,q)dtds\ \ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(m,n)\,K_{\alpha_1,\alpha_2}(m,n,p,q)dmdn]\}$

By using change of variable

$= e^{-\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)}\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(m,n)\,K_{\alpha_1,\alpha_2}(m,n,p,q)dmdn[A_1\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,s)\,K_{\alpha_1,\alpha_2}(t,s,p,q)dtds$

$$+A_2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(t,s)\, K_{\alpha_1,\alpha_2}(t,s,p,q)dtds]$$

$$= e^{-\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)}\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(m,n)\, K_{\alpha_1,\alpha_2}(m,n,p,q)dmdn$$
$$\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[A_1 f(t,s) + A_2\, g(t,s)]K_{\alpha_1,\alpha_2}(t,s,p,q)dtds\}\text{----}(2)$$

From (1) and (2) result (i) is proved.
Similarly we can proved result (ii)

$$h * (A_1 f + A_2 g) = A_1(h * f) + A_2(h * g).$$

3.2. Shifting Property

Prove that (i) $(\alpha f * g) = \alpha(f * g)$
(ii) $(f * \alpha g) = \alpha(f * g)$

Proof-
Consider,
$LHS = (\alpha f * g)$

By using (3.1) and (3.2)

$$= e^{-\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)}F_{\alpha_1,\alpha_2}\{\alpha f(t,s)\}F_{\alpha_1,\alpha_2}\{g(m,n)\}$$

$$= e^{-\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{\alpha f(t,s)\}\, K_{\alpha_1,\alpha_2}(t,s,p,q)dtds\ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{g(m,n)\}\, K_{\alpha_1,\alpha_2}(m,n,p,q)dmdn$$

$$= e^{-\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)}\alpha\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{f(t,s)\}\, K_{\alpha_1,\alpha_2}(t,s,p,q)dtds\ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{g(m,n)\}\, K_{\alpha_1,\alpha_2}(m,n,p,q)dmdn$$

$$= e^{-\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)}\alpha F_{\alpha_1,\alpha_2}\{f(t,s)\}F_{\alpha_1,\alpha_2}\{g(m,n)\}$$

$$= \alpha(f * g)$$

Similarly we can proved result (ii)
$(f * \alpha g) = \alpha(f * g)$

3.3 Distributive Property

Prove that $f * (g + h) = (f * g) + (f * h)$

Proof-
Consider,
$LHS = f * (g + h) \quad = f * \omega \qquad (\because \omega = g + h)$
By using (3.1) and (3.2)
$LHS = f * \omega$

$$= e^{-\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)}F_{\alpha_1,\alpha_2}\{f(t,s)\}F_{\alpha_1,\alpha_2}\{\omega(m,n)\}$$
$$= e^{-\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{f(t,s)\}\, K_{\alpha_1,\alpha_2}(t,s,p,q)dtds\ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{\omega(m,n)\}\, K_{\alpha_1,\alpha_2}(m,n,p,q)dmdn$$
$$= e^{-\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{f(t,s)\}\, K_{\alpha_1,\alpha_2}(t,s,p,q)dtd\ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{(g+h)(m,n)\}\, K_{\alpha_1,\alpha_2}(m,n,p,q)dmdn$$
$$= e^{-\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{f(t,s)\}\, K_{\alpha_1,\alpha_2}(t,s,p,q)dtds\ \{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{g(m,n)\}\, K_{\alpha_1,\alpha_2}(m,n,p,q)dmdn$$
$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{h(m,n)\}\, K_{\alpha_1,\alpha_2}(m,n,p,q)dmdn\}$$
$$= e^{-\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)}\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{f(t,s)\}\, K_{\alpha_1,\alpha_2}(t,s,p,q)dtds\ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{g(m,n)\}\, K_{\alpha_1,\alpha_2}(m,n,p,q)dmdn\}$$
$$+\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{f(t,s)\}\, K_{\alpha_1,\alpha_2}(t,s,p,q)dtds\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(m,n)\}\, K_{\alpha_1,\alpha_2}(m,n,p,q)dmdn\}$$
$$= (f * g) + (f * h)$$

3.4 Associative Property

Prove that-

$(f * g) * h = f * (g * h)$

Proof-

Consider

$LHS = (f * g) * h \; = \delta * h \qquad (\because \delta = f * g)$

By using (3.1) and (3.2)

$$= e^{-\frac{i}{2}(p^2 cot\emptyset_1 + q^2 cot\emptyset_2)} F_{\alpha_1,\alpha_2}\{\delta(t,s)\} F_{\alpha_1,\alpha_2}\{h(m,n)\}$$
$$= e^{-\frac{i}{2}(p^2 cot\emptyset_1 + q^2 cot\emptyset_2)} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{\delta(t,s)\} K_{\alpha_1,\alpha_2}(t,s,p,q)dtds \; \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{h(m,n)\} K_{\alpha_1,\alpha_2}(m,n,p,q)dmdn$$
$$= e^{-\frac{i}{2}(p^2 cot\emptyset_1 + q^2 cot\emptyset_2)} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{(f * g)(t,s)\} K_{\alpha_1,\alpha_2}(t,s,p,q)dtds \; \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{h(m,n)\} K_{\alpha_1,\alpha_2}(m,n,p,q)dmdn$$
$$= e^{-\frac{i}{2}(p^2 cot\emptyset_1 + q^2 cot\emptyset_2)} \{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[e^{-\frac{i}{2}(p^2 cot\emptyset_1 + q^2 cot\emptyset_2)}(F_{\alpha_1,\alpha_2} f(t,s)) \; (F_{\alpha_1,\alpha_2} g(t,s)) K_{\alpha_1,\alpha_2}(t,s,p,q)dtds] \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{h(m,n)\} K_{\alpha_1,\alpha_2}(m,n,p,q)dmdn$$
$$= e^{-\frac{i}{2}(p^2 cot\emptyset_1 + q^2 cot\emptyset_2)} e^{-\frac{i}{2}(p^2 cot\emptyset_1 + q^2 cot\emptyset_2)} \{ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \; [\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,s) K_{\alpha_1,\alpha_2}(t,s,p,q)dtds \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(m,n)) K_{\alpha_1,\alpha_2}(m,n,p,q)dmdn] \; K_{\alpha_1,\alpha_2}(t,s,p,q)dtds\} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(m,n)\} K_{\alpha_1,\alpha_2}(m,n,p,q)dmdn$$

$$= e^{-\frac{i}{2}(p^2 cot\emptyset_1 + q^2 cot\emptyset_2)} e^{-\frac{i}{2}(p^2 cot\emptyset_1 + q^2 cot\emptyset_2)} \{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \; [\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,s) K_{\alpha_1,\alpha_2}(t,s,p,q)dtds] \; K_{\alpha_1,\alpha_2}(t,s,p,q)dtds\} \; [\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(m,n)) K_{\alpha_1,\alpha_2}(m,n,p,q)dmdn \; \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(m,n)\} K_{\alpha_1,\alpha_2}(m,n,p,q)dmdn]$$

$$= e^{-\frac{i}{2}(p^2 cot\emptyset_1 + q^2 cot\emptyset_2)} e^{-\frac{i}{2}(p^2 cot\emptyset_1 + q^2 cot\emptyset_2)} \; \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,s) K_{\alpha_1,\alpha_2}(t,s,p,q)dtds] \; [\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(m,n)) K_{\alpha_1,\alpha_2}(m,n,p,q)dmdn \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(m,n)\} K_{\alpha_1,\alpha_2}(m,n,p,q)dmdn] \; K_{\alpha_1,\alpha_2}(t,s,p,q)dtds \; = f * (g * h)$$

3.5 Conjugation Property

Prove that-

$(\overline{f * g}) = \bar{g} * \bar{f}$

Proof-

By using (3.1) and (3.2)

$$f * g = e^{-\frac{i}{2}(p^2 cot\emptyset_1 + q^2 cot\emptyset_2)} F_{\alpha_1,\alpha_2}\{f(t,s)\} F_{\alpha_1,\alpha_2}\{g(m,n)\}$$
$$= e^{-\frac{i}{2}(p^2 cot\emptyset_1 + q^2 cot\emptyset_2)} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{f(t,s)\} K_{\alpha_1,\alpha_2}(t,s,p,q)dtds \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{g(m,n)\} K_{\alpha_1,\alpha_2}(m,n,p,q)dmdn$$
$$\overline{f * g} = \left(\sqrt{\frac{1 + icot\emptyset_1}{2\pi}} \sqrt{\frac{1 + icot\emptyset_2}{2\pi}}\right)^2 e^{\frac{i}{2}(p^2 cot\emptyset_1 + q^2 cot\emptyset_2)} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,s) g(m,n) \; e^{-\frac{i}{2}[(t^2+p^2)cot\emptyset_1 + (s^2+q^2)cot\emptyset_2]} \; e^{i[tpcosec\emptyset_1 + sqcosec\emptyset_2]} \; e^{-\frac{i}{2}[(m^2+p^2)cot\emptyset_1 + (n^2+q^2)cot\emptyset_2]} \; e^{i[mpcosec\emptyset_1 + nqcosec\emptyset_2]} dmdndtds$$
$$= \left(\sqrt{\frac{1 + icot\emptyset_1}{2\pi}} \sqrt{\frac{1 + icot\emptyset_2}{2\pi}}\right)^2 e^{\frac{i}{2}(p^2 cot\emptyset_1 + q^2 cot\emptyset_2)} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,s) \; e^{-\frac{i}{2}[(t^2+p^2)cot\emptyset_1 + (s^2+q^2)cot\emptyset_2]} \; e^{i[tpcosec\emptyset_1 + sqcosec\emptyset_2]} \; dtds \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(m,n) \; e^{-\frac{i}{2}[(m^2+p^2)cot\emptyset_1 + (n^2+q^2)cot\emptyset_2]} \; e^{i[mpcosec\emptyset_1 + nqcosec\emptyset_2]} dmdn$$

$$= \bar{g} * \bar{f}$$

Hence proved.

BASIC PROPERTIES OF QUATERNION CONVOLUTION

1	**Linearity Property**	(i) $(A_1 f + A_2 g) * h = A_1(f * h) + A_2(g * h)$ (ii) $h * (A_1 f + A_2 g) = A_1(h * f) + A_2(h * g)$
2	**Shifting Property**	(i) $(\alpha f * g) = \alpha(f * g)$ (ii) $(f * \alpha g) = \alpha(f * g)$
3	**Distributive Property**	$f * (g + h) = (f * g) + (f * h)$
4	**Associative Property**	$(f * g) * h = f * (g * h)$
5	**Conjugation Property**	$(f * g) = \bar{g} * \bar{f}$

CONCLUSION

In this paper we have developed the new convolution structure of Fractional Quaternion Fourier Transform which is useful in image processing. Also some basic properties of quaternion convolution are proved.

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